Nicos Starreveld
Korteweg-de Vries Institute
University of Amsterdam
n.j.starreveld@uva.nl

Raf Bocklandt
Korteweg-de Vries Institute
University of Amsterdam
r.r.j.bocklandt@uva.nl

Interview Sir Roger Penrose

A passion for patterns, puzzles and physics

Sir Roger Penrose is a British mathematical physicist, mathematician and philosopher of science. He is Emeritus Rouse Ball Professor of mathematics at the University of Oxford. In July he visited Utrecht for the '19th UK and European Meeting on the Foundations of Physics' and on this occasion Nicos Starreveld and Raf Bocklandt, editors of this journal, had an interview with him about life, the universe and everything.

Penrose was born in 1931 in Colchester, Essex, England. From a young age he got familiar with mathematics and physics. His father, Lionel Penrose was a medical geneticist, he was a professor of eugenics at University College London. He also had a keen interest into mathematics and chess, interests that he transmitted to his sons. Penrose's mother, Margaret Leathes was also medically trained, but she did not practice medicine.

"My father had a deep interest in mathematics, we had long walks talking about nature and mathematics. The physics side came more from my brother Oliver who was very well read. He used to read to me the book *Mr. Tompkins in Wonderland*."

(Mr. Tompkins is a fictional character created by the physicist George Gamow. In this book he explores a dream where the speed of light and the Planck constant are different so that you can see relativistic and quantum effects with your own eyes.)

Penrose's older brother Oliver Penrose became a theoretical physicist and was a Professor of Mathematics at Heriot-Watt University in Edinburgh from 1986 until his retirement in 1994. His younger brother, Jonathan Penrose, is a chess grandmaster who won the British Chess Championship ten times between 1958 and 1969.

"I was at school, about fifteen years old, and I had to decide which subjects I wanted to choose for my final two years. When the headmaster asked me, I told him I wanted to do biology, mathematics and chemistry. He told me it was not possible to do mathematics and biology together. I then said mathematics, physics and chemistry. My parents were very annoyed when I got home, they wanted me to become a doctor."

Instead Penrose started undergraduate studies in mathematics at University College London, where he graduated with a first class degree. Afterwards he moved to Cambridge to do research with Prof. W.V.D. Hodge, who was a professor in geometry and astronomy. In 1958 he completed his PhD, with his doctoral thesis *Tensor Methods in Algebraic Geometry*, written under the supervision of Prof. John Todd.

Moving to physics

Although mathematics was what he loved the most, physics never ceased to excite.

"I was interested in physics all the time. The thing that set me off in that direction was a series of radio talks by Fred Hoyle (British astronomer and cosmologist in Cambridge University). The last talk was about the steady-state theory. In this theory he described how the universe remains the same and how galaxies disappear when they reach the speed of light. I did not understand that, how could they disappear like that? They fade away maybe, so I went to Cambridge to discuss it with my brother Oliver.

We were having lunch together and he said 'I am no expert in that, but a good friend of mine is sitting over there, he will tell you, he is an expert in cosmology.' That was Dennis Sciama, so I described this to him and he had never seen this argument, he was impressed enough. When I went to Cambridge to do research in pure mathematics we had discussions about cosmology and physics, he knew everything that was going on. And this was wonderful education for me."

The discussions with cosmologist Dennis Sciama inspired Penrose to make the switch to cosmology; he started doing research in geometry and general relativity. In the years that followed, while he held a position at the department of mathematics at Birkbeck college in London, Penrose developed novel mathematical methods in general relativity. He introduced the calculus of spinors and showed that this method, when compared to the classical method of tensors, yields simpler equations when solving Einstein's equations. This path led Penrose to a much deeper understanding of the laws of the universe.


Photo: Wikimedia Commons, Biswarup Ganguly

Sir Roger Penrose

In 1964 Penrose wrote the epoch-making paper where he stated the First Singularity Theorem. In this paper he introduced the concept of a 'trapped surface' to characterize a gravitational collapse that has reached a point of no return. Gravitational collapse describes the contraction of a massive astronomical object, under the influence of its own gravity, towards its center of gravity. Gravitational collapse is the process that gives birth to black holes! The 'trapped surface' describes the region around a black hole where light is 'trapped' and can't escape out of it any more. The formation of black holes as a result of gravitational collapse is described by Einstein's general relativity, Penrose's First Singularity Theorem states that this process will inevitably lead to the formation of a singularity. Penrose still remembers the precise moment these ideas came to him.

"I was at Birkbeck College at that time and a friend of mine, Ivo Robinson, was visiting me. He was somebody who liked talking, he spoke very elegantly and was very nice to listen to. He was talking, talking, talking to me while we were walking along the street.

Then we got to a road where we had to stop talking because of the traffic. As I crossed the road, an idea came to me but on the other side he started talking to me again and the idea locked up into my head.

Later that day he went away and I was thinking 'why do I feel this state of elation?' I felt happy about something, but I didn't know what it was. So I went through the day: what kind of breakfast I had,... all things that had happened, one after the other.

Then I remembered crossing the street and suddenly it came back. I realized what I thought of: a characterization of the collapse in a way that has no symmetries at all. It's basically light rays converging all around on a sphere, so you have a topological condition. It was just a rough idea: I didn't have a complete working argument, it was still slightly uncomfortable in some places, but the main elements were all there."

At the same time Penrose's work on space-time singularities was inspiring Dennis Sciama's brilliant student, Steven Hawking, who wrote the magnificent essay entitled *Singularities and the Geometry of space-time*. Together, Hawking and Penrose proved the gravitational singularity theorems and made the theoretical prediction that black holes emit radiation, called Hawking radiation. The first meeting between Hawking and Penrose is also portrayed in 'The Theory of Everything' but he warns us to take that movie with a grain of salt.

"Don't believe the film! The film has Stephen going to London to hear me giving a lecture. It shows Stephen listening to it and afterwards coming at me, saying he was heavily inspired by this lecture.

I did give a talk in London, but Stephen Hawking was not there, only Dennis Sciama was. This talk was about the singularity in the black hole collapse using topology. So Dennis asked me to give a repeat talk in Cambridge, which I did and then I did meet Stephen. We discussed things and this started him off on this kind of thinking."

Art and Escher

Apart from his work in mathematics and physics, Penrose is also known for some artistic creations as well. His Penrose tiles adorn the floors and walls of many buildings all over the world and the famous impossible triangle was first described in an psychology article by himself and his father. Moreover they both corresponded with M.C. Escher and inspired some of his artwork.

"In 1954, I was in my second year as a graduate student in Cambridge doing mathematics and the International Congress of Mathematicians that year was to be held in Amsterdam. At one point during the conference, I was getting onto a bus and somebody else was just getting off.

This was one of the lecturers at Cambridge, Sean Wylie. He taught topology, which I liked a lot. He was holding a catalog with the picture *Night and Day*, the one with the birds flying in both directions. I asked: 'What is that?' He replied: 'Oh, there is this exhibition with art from an artist called Escher and I think you will be interested.'

I had never heard of him but I went to the Van Gogh Museum and there were these pictures all around the room. Absolutely stunning, I was blown of my feet. It was quite a big room, and several of them stuck in my mind. Particularly the one called *Relativity*, with the stairs in different directions.

I came away from that thinking: 'Gosh, this is amazing. I'd like to try something myself.' Something new, just an idea maybe that I hadn't seen. So I played around with rivers and bridges, roads going in different directions, an impossible situation. I simplified it to this triangle, a tripod triangle. When I came home and I showed it to my father and he asked: 'What's that?' He also got caught by this and he produced some buildings which were inconsistent and then he made an impossible staircase.

We decided we like to write a paper on it, but we didn't know what the subject was and what journal we would submit it to. My father knew the editor of the *British Journal of Psychology* and he said: 'Well, we'll call it psychology and we'll send it to that Journal.' They accepted it and in this paper we gave credit to Escher and to the catalog of the museum.

My father sent a copy of the article to Escher — I don't know if he had this address I think it went through somebody else. Anyway, it did find his way to Escher and he started to communicate with my father."

The impossible triangle

Later Penrose also had the opportunity to meet with Escher in person.

"I was very bad at correspondence but once I was driving in Holland with my wife and we passed Baarn, where Escher lived. I was given his phone number from my father's letters. I called Escher and he was very friendly and said: 'Come and have tea.'

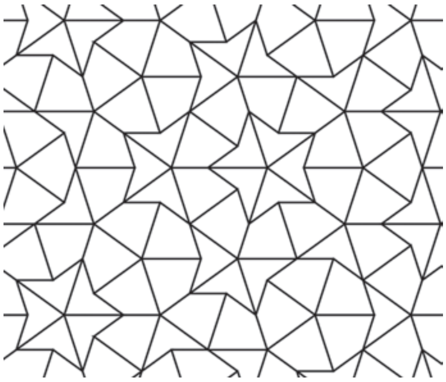
We went there and he was very generous and pleasant. There was this long table almost completely empty except some piles of prints at the end. He said: 'This is where I work.' I was expecting staircases going out of the windows in all directions, but no it was just a very ordinary house. Then he said: 'Look I have this pile of prints. Of these ones I've not got very many left, so I can't give you one of them. But from that pile over there you can take one.' So, I took the pile and went through it, some of them were quite familiar to me, but then there was one I hadn't seen before.

I said to him 'This one', and he replied: 'That's very interesting, usually people don't appreciate that picture.' It's called *Fishes and Scales*. It's a square but slightly rounded corners and you have a big fish on the bottom left. Its scales soon begin to become other fish and if you follow them then you end up at another big fish. Its scales are fish too and as you go back they become the fish you started with."

That was not the end of Penrose's relationship with Escher. Later he would also be the source of inspiration of one of his last paintings.

"When I visited him he gave me this print and all I could do was give him some cardboard pieces of a puzzle in return. These were all identical shapes which would tile a plane but in a way which required twelve different orientations before it repeated.

He then wrote to me to say: 'Look, I found out how it's done. Is this really



An example of a Penrose tiling

unique and what is the plan underlying it?' So I showed him the plan: you match this against, this against this, et cetera. You can have other designs like this and I drew some bird shape. I said it's not very good I'm sure you could do far better than this.

So he wrote back to me and he showed me the picture with the ghosts. He was not all together pleased with it. I think he thought he could have done better but it was probably the one of last ones he made."

This puzzle would later evolve into the famous Penrose tilings that tile the entire plane in a nonperiodic way. Surely Escher would have loved this but unfortunately he passed away before he could play around with it.

"I would say I was too slow to find these things. It was only just shortly after he died. My father died around the same time and both of them would have got a lot out of them."

Another set of pictures by Escher that are very interesting to mathematicians are his tilings of the hyperbolic plane. The hyperbolic plane is an abstract two-dimensional space that behaves in almost all respects as the ordinary Euclidean plane except that you can draw more than one line through a point parallel to a given point. It is impossible to embed the hyperbolic plane into ordinary Euclidean space in such a way that it preserves all distances, but there are other interesting depictions of it. In the Poincaré disc model all the points of the hyperbolic plane are mapped into a disk and all lines in the hyperbolic plane become circle segments that stand perpendicular on the boundary of the disk. This model has the special property that it is conformal, which means that it preserves angles: if two lines make a certain angle in the hyperbolic plane the corresponding circle segments in the Poincaré disk will make the same angle. Escher used this model for his 'Circle limit' paintings. Although the model preserves angles it does not preserve size and this is why the fishes, angles and demons get smaller and smaller as they approach the limit circle, while in the hyperbolic plane all these creatures have exactly the same size.

"These pictures came about as a result of the same conference in Amsterdam. Another mathematician at this conference was Coxeter. He was a very distinguished geometer and he was very impressed by Escher. He said to him: 'Look there you could do very good things with hyperbolic geometry, and particularly the Beltrami model.' You have to be careful about this, because most people call it the Poincaré disk. But the history of it is not quite so straightforward. Poincaré did rediscover it but originally it was discovered by Beltrami, an Italian geometer.

Beltrami had discovered the conformal representation of the hyperbolic plane, which is the Poincaré disk, the projective representation, which is called the Klein representation, and the half plane representation, which is called the Poincaré half plane. He had found all these models together and he had figured out the relationship between them. This was very cleverly done but other people didn't know about it. Poincaré did other things with it which are important, but Beltrami had put the whole thing in a much better geometrical picture.

The conformal model is the most beautiful one and that is the one Coxeter suggested Escher should use. And that he did, he made an extraordinarily precise picture. Right up to the edge you have these tiny figures and they're still very correct."

Conformal cosmology

Penrose's fascination for conformal symmetries also shines through in his work on cosmology. He developed a new theory in which the universe goes through an endless sequence of cycles, that each start with a big bang. But unlike other theories in which each cycle ends with a crunch, Penrose theory predicts that the universe keeps on expanding and expanding. But as the last stars and galaxies fade away and all the black holes evaporate, it becomes conformally symmetric. This means that there is no difference between big and small, only angles are preserved. From a conformal point of view this vast radiation filled universe looks exactly the same as the tiny concentrated energetic soup that we see just after the big bang and the cycle can start all over again.

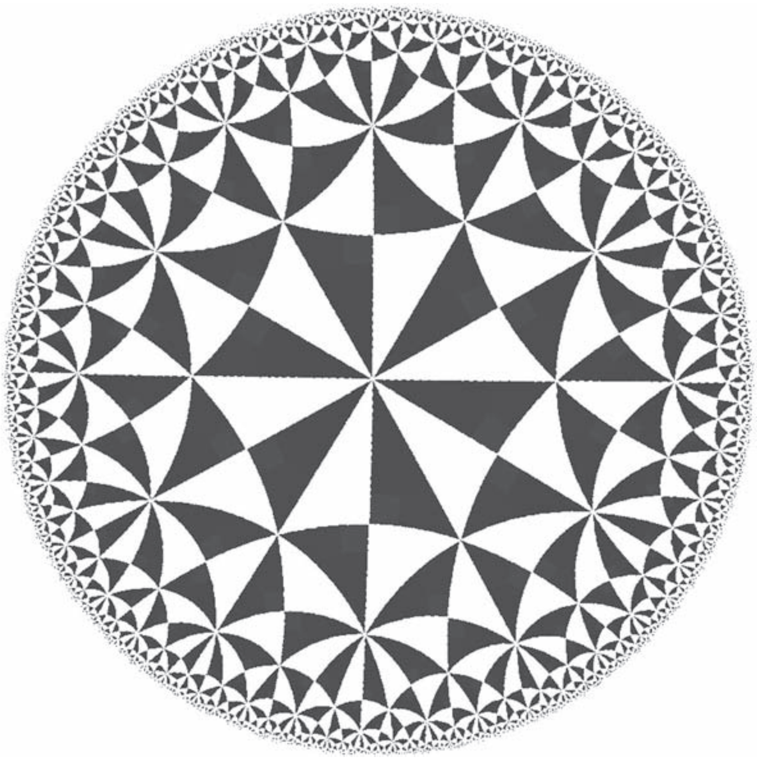
"One of the things that deeply troubled me for many many decades is the second law of thermodynamics. Excuse me when I call that the mammoth in the room. People say the elephant in the room but it is even bigger. The second law of thermodynamics tells us that things get more random so to speak: entropy is increasing. Saying exactly the same thing the other way around, back in the past things get less and less random.

Now go back until you see the earliest things we can observe, the Cosmic Microwave Background. If you look at it, one of the most striking features of that is its thermal spectrum. If you compare it to the spectral curve of a radiating black body, it fits absolutely beautifully. It tells you that you have a state of maximum entropy. Back in time the entropy gets smaller but you get to a maximum.

That's strange... but it's not a paradox because you have not taken gravity into consideration. Gravity tends to work the other way: entropy goes up when things clump whereas for ordinary gas in a box entropy goes up as it spreads out. Gravity being universally attractive means that you have a uniform distribution with low entropy and then the entropy goes up when it condenses. Well that is why we see galaxies, stars, our Sun..."

But this does not explain why the universe started off in this thermal equilibrium. The standard way cosmologists solve this puzzle is by introducing inflation. This theory proposes that just after the big bang the universe went through a stage of very rapid expansion, in which it grew exponentially by a factor of 10^{50}. So everything we see around us was so close together that before inflation started it all had the same temperature and therefore the cosmic background radiation looks uniform in all directions. But Penrose is not convinced.

"Inflation is just not a solution! The easiest way to think of it is when you reverse time. Imagine your universe and as it contracts, the irregularities will increase and then they will produce black holes. You can put the inflaton field in as much as you like, it makes no difference. That irregular start will not be smoothened by inflation. It's like a fractal: if you stretch it out it gets worse."



Coxeter's tiling of the hyperbolic plane that was an inspiration for Escher's 'Circle limits' paintings

Instead Penrose came up with a different way to solve the problem.

"Some years ago, I had this idea of conformal cyclic cosmology. In this model you have a universe where the Big Bang is the conformal continuation of the exponential expansion caused by Einstein's cosmological constant. The cosmological constant ensures that the universe will expand exponentially in the far future. In a way it does the same as inflation but at the wrong end of the universe: not at the beginning but at the end.

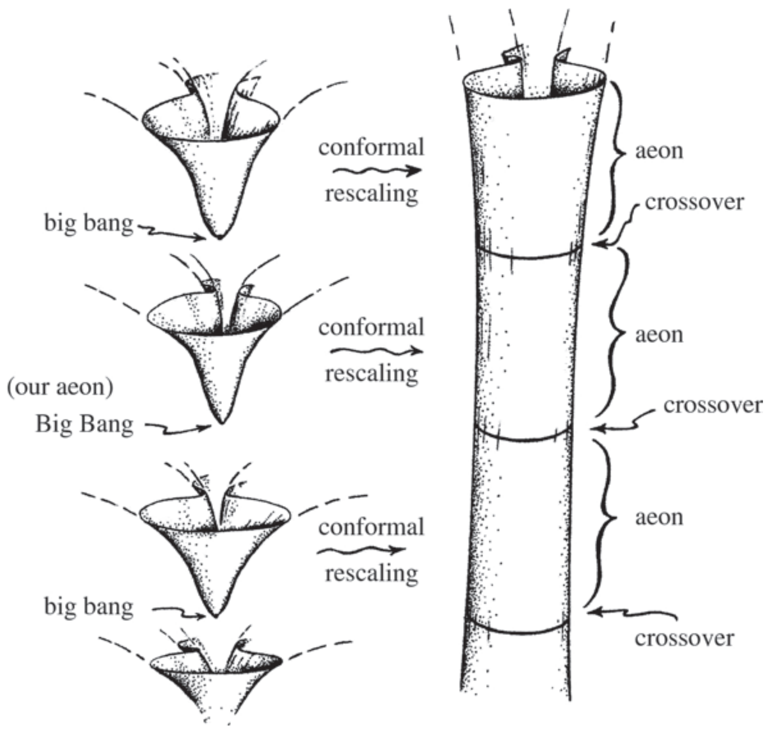
However, if you have a conformal picture and you look at our Big Bang conformally, you can stretch it out and it looks very similar to the other end of the universe. So my contention is that they actually match and that our Big Bang was the conformal continuation of the remote future of a previous universe."

Penrose proposal has a strange consequence. If the universe goes through consecutive cycles the second law of thermodynamics tells us the entropy should be increasing all the time. Yet in his model each cycle the universe seems to start with the same entropy. How does this work?

"You mean why you don't have entropy increasing, increasing, increasing,... with each cycle? The reason is as follows. Where is the most entropy in our universe now? Easily in supermassive black holes. What happens to those supermassive black holes? Well they evaporate away and with them some of the degrees of freedom in the universe get lost. To estimate the entropy, you have to have a phase space. But if you lose degrees of freedom this means your phase space collapses down.

You say, okay that black hole has swallowed all those degrees of freedom. It has disappeared, forget it. But up to that point I had been counting those degrees of freedom in my calculation of entropy. When that black hole is gone I change my mind about what I regard as the phase space. I say okay, with this gone, my measure of entropy has come down. It's not that it has violated the second law. It's just that I have a new renormalized definition of entropy, which does not include those degrees of freedom. That new quantity, which I now call the entropy, is smaller.

Once you've lost all the black holes, the entropy value comes down because we are not interested in those degrees of freedom anymore. The second law starts with this new renormalized entropy and that's the argument. It's a bit subtle, but I think it's consistent: you can live with the second law, as well as with cyclic cosmology."



Schematic description of conformal cyclic cosmology, as appeared in Penrose's article 'The big bang and its dark-matter content: whence, whither, and wherefore', publiched in *Foundations of Physics* 48(10), October 2018, 1177–1190.

But is all this just pure speculation or does he have evidence for his theory?

"Just a few days ago, I was in Poland visiting my colleague Krzysztof Meissner. He and another astronomer, Daniel An, had done an analysis based on my idea of conformal cyclic cosmology and they found evidence for this which seems to be extremely significant.

They were seeing little points in the sky which seemed to emit huge amounts of energy. When Meissner mentioned this, I thought: 'Yeah, I think I know what that is.' In the previous eon before the Big Bang you will have supermassive black holes and these black holes will after a while start to radiate away by Hawking radiation. This is after a very long time, when the universe has gotten much colder. You have to wait 10^{100} years, an enormous time, but then they start to radiate and the entire energy of these black holes is concentrated into one point in the sky.

We have just written a paper about this, which we have sent to Physical Review Letters. In it we identify twenty points in the sky where there is a source of a huge amount of energy. We call these Hawking points. Although we're little cautious to say what they are, we argue that they are remnants of a previous cosmological cycle."

So even at the age of 86 Penrose is still actively exploring and trying to understand the cosmos. Whether his proposal of cyclic conformal cosmology will work out, time will tell. Lot's of time, perhaps even 10^{100} years...

Event 22 January 2019
In January 2019 Sir Prof. Roger Penrose will be in the Netherlands for the presentation of the Dutch translation of his monumental book *The Road to Reality: A Complete Guide to the Laws of the Universe*. On 22 January the Institute of Advanced Study of the University of Amsterdam will host a public event where Penrose will discuss the three world model. The three world model was proposed by Penrose as a means to understand how the physical, mental and mathematical worlds are connected. The book will be published by Merlin Uitgaven.